# PROCEEDINGS

OF THE

# ROYAL SOCIETY OF EDINBURGH.

VOL. VIII. 1873–74. No. 87.

## Ninety-First Session.

*Monday, 24th November* 1873.

Sir ROBERT CHRISTISON, Bart., President, in the Chair.

The following Council were elected :—

*President.*
Sir WILLIAM THOMSON, Knt., LL.D.

*Honorary Vice-Presidents.*
His Grace the DUKE of ARGYLL.
Sir ROBERT CHRISTISON, Bart., M.D.

*Vice-Presidents.*

Principal Sir Alex. Grant, Bart.
Sir W. Stirling-Maxwell, Bart.
David Milne Home, LL.D.
Professor Kelland.
Rev. W. Lindsay Alexander, D.D.
David Stevenson, Esq., C.E.

*General Secretary*—Dr John Hutton Balfour.

*Secretaries to Ordinary Meetings.*
Professor Tait.
Professor Turner.

*Treasurer*—David Smith, Esq.
*Curator of Library and Museum*—Dr Maclagan.

*Councillors.*

Professor A. Dickson.
James Leslie, Esq., C.E.
Rev. Thomas Brown.
James Dewar, Esq.
Professor Lister.
George Robertson, Esq.
Captain T. P. White.
The Hon. Lord Neaves.
The Right Rev. Bishop Cotterill.
Professor A. Crum Brown.
Dr Arthur Mitchell.
George Forbes, Esq.

*Monday, 1st December* 1873.

SIR ROBERT CHRISTISON, Bart., Honorary Vice-President, in the Chair.

The following Communications were read:—

## 1. Laboratory Notes. By Professor Tait.

### 1. First Approximation to a Thermo-electric Diagram.

(This Paper will appear in the Transactions of the Society.)

### 2. On the Flow of Water through Fine Tubes.

Dr Matthews Duncan recently asked me whether the flow of blood under given pressure would be affected by a considerable change of *form* of the section of a small vein or artery. It appeared obvious, from the well-known experiments of Poiseuille (which show that when the bore of a capillary tube of circular section is sufficiently small, the flow through it is as the fourth power of the diameter), that the flow through a capillary tube of elliptical section must be less than that through a circular tube of equal sectional area. The accepted theory of fluid friction might enable us to obtain a solution for an elliptic tube, but the assumptions requisite for its deduction appear extremely unlikely to be fulfilled in practice, so that I asked Messrs C. G. Knott and C. M. Smith to make some direct experimental comparisons between various circular and elliptic tubes, specially drawn for the purpose, and of the same material. The present preliminary experiments, unfortunately, refer only to tubes the smallest of which has nearly the bore of the largest of those used by Poiseuille.

The tubes were carefully calibrated and the worst rejected. A length of twenty inches was cut from the most uniform portion of each of the selected tubes, and the axes of the section (when elliptical) were carefully measured at each end. This determination was checked by weighing the column of mercury employed for calibration. Water, at a fixed temperature, was drawn under

fixed pressure, for a given time, through each, and its quantity measured. The following are the experimental results:—

| Section. | Weight of 1 inch Hg. | Efflux per min. | T. | P. | Axes of sections in hundredths of an inch. | Ratio of axes. | Cal. weight of 1 inch. Hg. |
|---|---|---|---|---|---|---|---|
| Elliptic. | | Cub. in. | C. | In. | | | |
| I. | ·249 | 3·30 | 9·4 | 23·65 | 8·5 } { 1·5<br>8·25 } { 1·8 | 5·073 | ·238 |
| II. | ·357 | 7·87 | 9·5 | 23·65 | 9· } { 2·3<br>9·2 } { 2·3 | 3·956 | ·359 |
| III. | ·441 | 12· | 10· | 23·57 | 9·4 } { 2·6<br>10·1 } { 2·7 | 3·679 | ·445 |
| IV. | ·685 | 21·77 | 10· | 23·57 | 12·75 } { 3·1<br>13· } { 3·2 | 4·087 | ·683 |
| Circular. | | | | | Diameters of ends. | | |
| I. | ·223 | 6·16 | 10· | 23·1 | 3·6 3·6 | ... | ·223 |
| II. | ·301 | 9·62 | 10· | 23·1 | 4·1 4·3 | ... | ·304 |
| III. | ·357 | 11·81 | 10· | 23·1 | 4·4 4·5 | ... | ·341 |
| IV. | ·646 | 22·6 | 9·9 | 23·1 | 6·2 6·2 | ... | ·661 |
| V. | 1·213 | 52·8 | 11·5 | 23·2 | 8·3 8·6 | ... | 1·229 |
| VI. | 1·632 | 67· | 11· | 23·2 | 9·6 9·6 | ... | 1·587 |

The two last were added with a view of finding the effect of still farther increasing the section. A comparison of the first and second groups of four shows the very considerable effect of the elliptic form in diminishing the rate of flow.

## 2. Note on the Transformation of Double and Triple Integrals. By Professor Tait.

1. If we have two equations of the form

$$f(u, v, \xi, \eta) = 0 ,$$
$$F(u, v, \xi, \eta) = 0 ,$$

$u$ and $v$ are given as functions of $\xi$ and $\eta$, or *vice versâ*. Here either $u$ and $v$, or $\xi$ and $\eta$, may be the ordinary Cartesian $x$ and $y$, or any given functions of them.

Now, if we write with Hamilton, since we are dealing with two independent variables only,

$$\nabla = i\frac{d}{dx} + j\frac{d}{dy} ,$$

we have

$$\nabla = \nabla u\frac{d}{du} + \nabla v\frac{d}{dv} = \nabla\xi\frac{d}{d\xi} + \nabla\eta\frac{d}{d\eta} \quad . \quad . \quad . \quad . \quad (1)$$

The proof may be easily given in a Cartesian form by operating by $Si$ and $Sj$ separately. For the former operation gives

$$\frac{d}{dx} = \frac{du}{dx}\frac{d}{du} + \frac{dv}{dx}\frac{d}{dv} = \frac{d\xi}{dx}\frac{d}{d\xi} + \frac{d\eta}{dx}\frac{d}{d\eta},$$

equations manifestly true.

2. Now, the elementary area included by the curves $u$, $u+\delta u$, $v$, $v+\delta v$, is easily seen to be

$$\frac{\delta u \delta v}{TV\nabla u\,\nabla v}.$$

Hence we have the following transformations of a double integral extended over a given area :—

$$\iint P\,dxdy = \iint P\,\frac{dudv}{TV\nabla u\nabla v} = \iint P\,\frac{d\xi d\eta}{TV\nabla\xi\nabla\eta}.$$

But by (1) we see at once that

$$TV\nabla\xi\,\nabla\eta = \begin{vmatrix} \frac{d\xi}{du}, & \frac{d\xi}{dv} \\ \frac{d\eta}{du}, & \frac{d\eta}{dv} \end{vmatrix} TV\nabla u\nabla v,$$

whence, of course, the general proposition

$$\begin{vmatrix} \frac{d\xi}{du}, & \frac{d\xi}{dv} \\ \frac{d\eta}{du}, & \frac{d\eta}{dv} \end{vmatrix} \begin{vmatrix} \frac{du}{d\xi}, & \frac{dv}{d\xi} \\ \frac{du}{d\eta}, & \frac{dv}{d\eta} \end{vmatrix} = 1,$$

and the common transformation

$$\iint P\,dxdy = \iint P \begin{vmatrix} \frac{dx}{du}, & \frac{dx}{dv} \\ \frac{dy}{du}, & \frac{dy}{dv} \end{vmatrix} dudv.$$

3. Dealing with triple integrals, $\nabla$ takes the ordinary Hamiltonian form, and an additional term is added to each of the members of (1), which thus at once gives us the mode of introducing $\nabla$ into any system of curvilinear cö-ordinates.

The element of volume included by the surfaces $u$, $u+\delta u$, $v$, $v+\delta v$, $w$, $w+\delta w$, is easily seen to be expressed by

$$-\frac{\delta u\delta v\delta w}{S.\nabla u\nabla v\nabla w}.$$

Hence we have the following—

$$\iiint \mathrm{P}\,dxdydz = -\iiint \mathrm{P}\,\frac{dudvdw}{S.\nabla u\nabla v\nabla w} = -\iiint \mathrm{P}\,\frac{d\xi d\eta d\zeta}{S.\nabla\xi\nabla\eta\nabla\zeta}.$$

From these we have, besides the more complex transformation from $u$, $v$, $w$, to $\xi$, $\eta$, $\zeta$, the common one

$$\iiint \mathrm{P}\,dxdydz = -\iiint \mathrm{P}\begin{vmatrix} \frac{dx}{du}, & \frac{dx}{dv}, & \frac{dx}{dw} \\ \frac{dy}{du}, & \frac{dy}{dv}, & \frac{dy}{dw} \\ \frac{dz}{du}, & \frac{dz}{dv}, & \frac{dz}{dw} \end{vmatrix} dudvdw,$$

and also the general theorem

$$\begin{vmatrix} \frac{d\xi}{du}, & \frac{d\xi}{dv}, & \frac{d\xi}{dw} \\ \frac{d\eta}{du}, & \frac{d\eta}{dv}, & \frac{d\eta}{dw} \\ \frac{d\zeta}{du}, & \frac{d\zeta}{dv}, & \frac{d\zeta}{dw} \end{vmatrix} \begin{vmatrix} \frac{du}{d\xi}, & \frac{du}{d\eta}, & \frac{du}{d\zeta} \\ \frac{dv}{d\xi}, & \frac{dv}{d\eta}, & \frac{dv}{d\zeta} \\ \frac{dw}{d\xi}, & \frac{dw}{d\eta}, & \frac{dw}{d\zeta} \end{vmatrix} = 1.$$

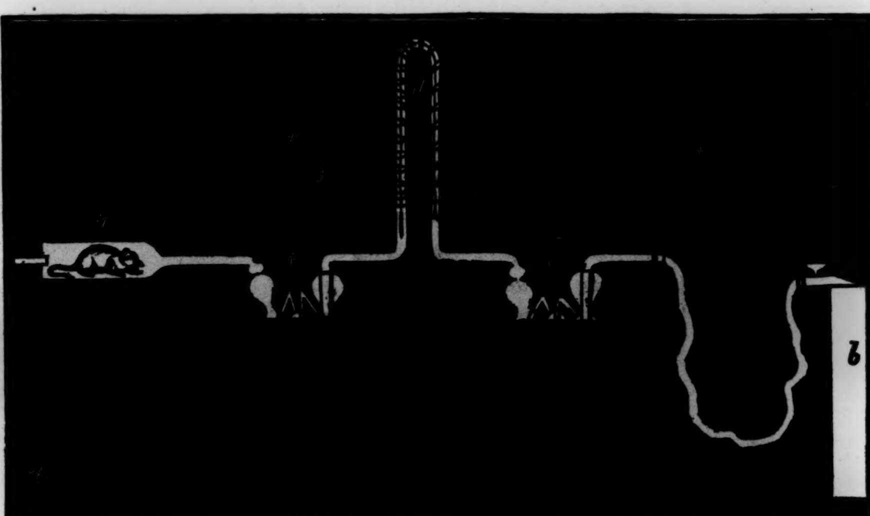
## 3. On the Physiological Action of Ozone. By James Dewar, Esq., Lecturer on Chemistry, and John G. M'Kendrick, M.D., Physiological Laboratory, University of Edinburgh.

A systematic investigation into the physiological action of ozone, so far as we are aware, has never been undertaken. Isolated observations have been made by many while engaged in the examination of its physical and chemical properties, which have chiefly tended to show that it acts as an irritant on the mucous membrane of the respiratory tract, and they have also observed the peculiar odour which it excites by its effect on the organ of smell, from which the name *ozone* originated. Beyond this little has been attempted.

Schönbein, indeed, showed * that a mouse imprisoned in an atmosphere of ozone died in about five minutes. From meteorological data, this observer also stated that the quantity of ozone in the atmosphere and the prevalence of epidemic diseases were in an inverse relation to each other both as to time and locality. This statement has probably given rise to the popular opinion that ozone not only acts as a powerful oxidising agent of decaying animal or vegetable matters, but also that it has a specific action on the animal body.

With the view of determining what action ozone exerts on the body, we commenced a series of experimental observations, which we now beg to lay before the Society.

1. *Mode of producing the Ozone* (see fig.).—The ozone in the following experiments was made by passing a current of dry air or




*Description of Figure.—a*, glass chamber for reception of the animal; *b*, gasometer; the current of air or gas passed from right to left of diagram; *c* (to the right), bulb-tube containing sulphuric acid; *c* (to the left), bulb-tube containing caustic potash or water; *d*, U tube; *e*, wire from — pole of induction coil continuous with platinum wire within the U tube; *f*, wire from + pole of induction coil continuous with copper wire coiled round U tube.

oxygen from a gasometer (*b*) through a narrow glass tube, bent for convenience like the letter U (*d*), about 3 feet in length, and containing a platinum wire 2 feet in length, which had been inserted

* British Association Reports, 1848.

into the interior of the tube, and one end (*e*) of which communicated with the outside through the wall of the tube. Round the whole external surface of this U-shaped tube a spiral of copper wire was coiled, and the induction current from a coil giving $\frac{1}{2}$-inch sparks was passed between the external copper (*f*) to the internal platinum wire (*e*), so as to have the platinum wire in the interior of the tube as the negative pole. After the current of gas was ozonised by the passage of the induction current, it was washed by passing through a bulb-tube (*c* to the left of the U tube) containing caustic potash when air was employed, or water when pure oxygen was used, in order to eliminate any traces of nitrous and nitric acids. To the right of the U tube another bulb-tube (*c*) was placed containing pure sulphuric acid, for removing aqueous vapour from the air, or gas passed through it. By means of the gasometer, the volume of gas passing through the apparatus could be ascertained.

2. *Method of Experiment.*—It was necessary, in the first place, to determine the action of ozone on the living animal imprisoned in an atmosphere containing a large proportion of ozone; and, in the second, to determine what action, if any, it exerted on the individual living tissues of the body.

Observations were made on frogs, birds, mice, rabbits, and on ourselves.

*Frogs.*—Numerous experiments were made on frogs, and the general effect on these animals is as follows:—About thirty seconds after introducing the animal into the chamber, through which a steady current of ozonised air was passing, the animal manifested symptoms of distress. The eyeballs were retracted, so as to be deeply sunk in the orbits, and the eyelids were firmly closed. It rubbed its nose occasionally with its fore paws. At first somewhat restless, the frog became lethargic, and the movements of respiration were reduced, both in frequency and force, to at least one-half the normal amount. On pushing the frog with a wire it might be excited to move, but usually it remained motionless. The position of the animal was peculiar—the neck arched, the head flattened, and it remained in a crouching attitude. This condition of lethargy has been observed to continue during a period of an hour and a half, at the end of which time the animal died. When

common air was introduced into the chamber instead of ozonised air, or if the frog was taken out of the chamber, it quickly recovered. These effects may be seen in the following experiment:—

A large healthy frog was introduced into the air-chamber, through which a current of air was passing sufficient to fill a litre flask in three minutes. At the end of two minutes, the respirations were 96 per minute. The induction machine was then set to work, so as to mix ozone with the air, the current passing through the chamber at the same rate. In half a minute the eyes were affected, and the respirations were reduced to 8 per minute. At the end of six minutes, the animal was quite motionless, and the respiratory movements had entirely ceased. Pure air was then introduced. In half a minute, there was a slight respiratory movement, and in eight minutes, the respirations numbered 85 per minute. At the end of other twelve minutes, ozone was again turned on, with the same results. The animal in this experiment was then subjected to atmospheres of common air and air mixed with ozone alternately, each period of immersion in the atmosphere consisting of ten minutes, with invariably the same effect. At the end of two hours it was removed from the chamber, and recovered.

In the case of the frog which died after being exposed to an atmosphere of ozonised air for an hour and a half, the heart was found pulsating after systemic death. It was full of dark-coloured blood. The lungs were slightly congested. In every part of the body the blood was in a venous condition.

In two experiments, frogs were exposed to the action, not of air mixed with ozone, but to a stream of oxygen mixed with ozone, and the results were somewhat different from those just narrated. The effects were not so well marked. When a frog was introduced into an atmosphere of pure oxygen, the animal was lively and vivacious, the eyes were wide open, and the respiratory movements were greatly accelerated. But when the oxygen contained a considerable quantity of ozone, the eyes were closed, the respiratory movements did not entirely cease, but were reduced from 100 or 110 to 8 or 12 per minute, and the creature was in a dormant condition. After exposure for a period of one hour, the web and the skin assumed a purple hue. After keeping the animal in such an atmosphere for $1\frac{3}{4}$ hour, it was in the same condition.

*Birds.*—A green linnet was put into the chamber, supplied with a strong current of air. At the end of five minutes, after the bird had become quiet, the respirations were 50 per minute. The air was then ozonised. In thirty seconds, the eyes were closed; in one minute, the respirations were reduced to 30 per minute; four minutes thereafter, the respiration was slow and gasping, and the number of movements 15 per minute; and in ten minutes, that is, fifteen and a half minutes after the introduction of ozonised air, the bird was dead. On opening the body, there was venous congestion of all the viscera. The lungs were of a dark purple colour, and showed a mottled appearance. The heart was still pulsating feebly. It was full of venous blood. The brain was pale. The blood corpuscles, when examined microscopically, were normal.

*Mammals.*—Several experiments were made on white mice and rabbits. With regard to mice, the general effects will be understood by detailing one experiment. A full-grown and apparently healthy white mouse was introduced into a vessel through which a stream of air was passing at the rate of 8 cubic inches per minute. Five minutes thereafter, the animal was evidently at ease, and the respirations were 136 per minute. The air was then ozonised. One minute after, the respirations were somewhat slower, but could not be readily counted, owing to the animal moving uneasily about and rubbing its nose with its fore paws. In four minutes from the time of introduction of the ozone, the respirations were 32 in a minute. The mouse now rested quietly, occasionally yawned, and when touched by a wire, moved, but always in such a direction as to place its head away as far as possible from the stream of ozonised air. At the end of fifteen minutes, the animal became excited, ran rapidly backwards and forwards, and then had a convulsive attack. It died, much convulsed, nineteen minutes after the introduction of the ozone. The body was colder than natural. There was venous congestion of all the abdominal viscera. The heart was still feebly pulsating, and the right auricle and ventricle were full of venous blood. The left side of the heart contained a small quantity of venous blood. The sinuses of the brain were full of dark blood, and the surface and base of the brain was traversed by vessels containing dark-coloured blood.

Two experiments were also made upon mice, in which, instead

of being supplied with ozonised air, they received ozonised oxygen. When a mouse breathed an atmosphere of pure oxygen, it became exceedingly active in its movements. It ran about examining every part of its prison, and breathed with such rapidity as to make it impossible to count the number of respirations taken during a minute. When the oxygen was ozonised, the mouse quickly showed the usual phenomena of the closed eyes and the reduction of the number of respirations, but it lived for a much longer period than in ozonised air. Instead of dying at the end of fifteen or twenty minutes after the introduction of the ozonised atmosphere, it lived for thirty-five or forty minutes. The number of respirations per minute became smaller, and the animal died in severe general convulsions. The blood, when examined quickly after death, has been found venous in all parts of the body. In both experiments, the temperature of the body was found to be much reduced.

As the reduced temperature of the body in these experiments might have been owing to the current of gas passing quickly over the bodies of the animals, two experiments were made, in which the glass air-chamber was immersed in a water-bath kept at a temperature of 30° C. The animals were supplied with atmosphere at the rate of 13 cubic inches per minute. The general results were the same as in the experiments made without the water-bath, but the temperature of the body on death was still below the normal.

Various experiments were also made on rabbits, with the same general results as in the case of mice. There was evident irritation of the eyes, causing closure of the lids, and the exudation from between their margins of a whitish fluid, probably lachrymal secretion. The respirations were reduced in number from 100 or 110 to from 36 to 30 per minute. In one experiment, only the head of the rabbit was introduced into a glass vessel, into which the stream of ozonised oxygen was transmitted so as to allow the experimenter to count by touch the number per minute of the pulsations of the heart. The result was, that immediately on the introduction of ozone the number of pulsations was much diminished, and the force of the contractions of the heart was so enfeebled that it could not be felt through the wall of the thorax. Still, in the bodies of rabbits killed in an atmosphere of ozonised air, or of

ozonised oxygen, the heart was found pulsating, and, as in the other cases, engorged with venous blood.

On breathing an atmosphere of ozonised oxygen ourselves, the chief effects observed were a suffocating feeling in the chest, a tendency to breathe slowly, an irritation of the back of the throat and of the glottis, and a tingling sensation, referred to the skin of the face and the conjunctivæ. The pulse became feebler. After breathing it as long as it was judicious to do, say for five or eight minutes, the suffocating feeling became stronger, and we were obliged to desist. The experiment was followed by violent irritating cough and sneezing, and for five or six hours thereafter by a sensation of rawness in the throat and air-passages.

The action of ozone on several of the chief physiological systems, and on various tissues, was also examined.

1. *On the Circulation.*—By a suitable apparatus, a frog was imprisoned in a chamber through which a stream of ozonised air, or of ozonised oxygen, passed, while at the same time the web was so placed under a microscope that the circulation in the smaller vessels and capillaries could be readily observed. The result was negative, inasmuch as no appreciable acceleration or retardation of the current of the circulation was seen.

2. *On the Reflex Action of the Spinal Cord.*—This function was not affected to any appreciable degree.

3. *On Muscular Contractility.*—By means of a myographion, the work done by the gastrocnemii of frogs, subjected to the action of ozone, was noted. The muscles were stimulated by a single opening or closing induction shock produced by Du Bois Reymond's apparatus and a Daniell's cell. The result was that the contractility and work-power of the muscle were found unaffected, as far as could be appreciated.

4. *On the Blood.*—When a thin layer of human blood on a slide is exposed to the action of ozone, the coloured corpuscles become paler, lose their definite outline, and if exposed for a period of five or ten minutes to the action of the current, they are dissolved, and a mass of molecular material is seen. The coloured corpuscles of the frog show, after the action of ozone, the formation of a nucleus. By prolonged exposure many of the nuclei apparently pass out of the substance of the corpuscle, numerous free nuclei are seen, and

some in the act of separating from the corpuscle have been observed. The colourless corpuscles are contracted into globular masses after the action of ozone. The general effects resemble those produced by a weak acid, such as very dilute acetic acid or a stream of carbonic acid.

5. *On Ciliary Motion.*—When the cilia of the common mussel (*Mytilus edulis*) were exposed to the action of ozone, while bathed in the fluid contained in the shell (sea-water), no effect was observed. This is owing to the protection to the cilia afforded by the water. If a very small amount of water covered the cilia, their action was at once arrested.

From the preceding experiments the following general facts may be stated:—

1. The inhalation of an atmosphere highly charged with ozone diminishes the number of respirations per minute.

2. The pulsations of the heart are reduced in strength, and this organ is found beating feebly after the death of the animal.

3. The blood is always found in a venous condition in all parts of the body, both in cases of death in an atmosphere of ozonised air and of ozonised oxygen.

4. Ozone exercises a destructive action on the living animal tissues if brought into immediate contact with them; but it does not affect them so readily if they are covered by a layer of fluid.

5. Ozone acts as an irritant to the mucous membrane of the nostrils and air-passages, as all observers have previously remarked.

At the present state of this inquiry, it would be premature to generalise regarding the relation between physiological action and the chemical properties of ozone; but we can hardly avoid pointing out that oxygen in this altered condition ($O_3 = 24$) is slightly denser than carbonic acid ($CO_2 = 22$), and that, although the chemical activity of the substance is much increased, yet when inhaled into the lungs, it must retard greatly the rate of diffusion of carbonic acid from the blood, which accounts for the venous character of that fluid after death. If, however, the physiological effect of ozone on respiration were merely due to its greater density, then we would expect its behaviour to be analogous to that of an atmosphere highly charged with carbonic acid. This has been found to be the case, more especially as regards the diminished number of respirations per minute, and the appearance of the blood after death.

If, however, this analogy were perfect, we would anticipate that the action of oxygen, partially ozonised, would not have produced death, as the amount of ozone in these experiments certainly did not exceed 10 per cent. As it was, all we have observed is that the animal only lives a somewhat longer time in ozonised oxygen than in ozonised air. We are thus induced to regard ozone as having some specific action on the blood, or in the reflex nervous arrangements of respiration, that future experiments may elucidate.

## 4. On a Compound formed by the addition of Bromacetic Acid to Sulphide of Methyl, and on some of its Derivatives. By Professor Crum Brown and Dr E. A. Letts.

(*Abstract.*)

The sulphine compounds discovered by v. Oefele, indicate that, notwithstanding the difference of atomicity, there exists an analogy between sulphur and nitrogen, these compounds corresponding to the salts of the ammonium bases. not only in chemical properties but also in physiological action.*

The research, the results of which are communicated in this paper, was undertaken with the view of examining this analogy in some other directions.

It seemed reasonable to suppose that, as the nitrile bases, such as trimethylamine and strychnia unite with chloracetic acid to form compounds such as hydrochlorate of betaine and of glycolyl-strychnia, the sulphides of the alcohol radicals should act in a similar way. Experiments show that this is the case—bromacetic acid acting readily on sulphide of methyl to form a beautifully crystallised compound to which the authors give the name of hydrobromate of methyl-thetine. Analyses proved this substance to have the composition corresponding to the formula $C_4H_9SBrO_2$ which is that of the sulphur analogue of the hydrobromate of betaine.

$$\underset{\text{hydrobromate of methylthetine.}}{Br-\overset{(CH_3)_2}{\overset{\|}{S}}-CH_2-COOH}\;;\;\underset{\text{hydrobromate of betaine.}}{Br-\overset{(CH_3)_3}{\overset{|||}{N}}-CH_2-COOH}$$

This view of its constitution is confirmed by its reactions.

* Brown and Fraser, "Proc. Royal Soc. Edin.," March 4th, 1872.

In addition to this substance, which served as a starting-point for the research, the nitrate, the chloroplatinate, the chloraurate, the bromaurate, and compounds formed by the action of the hydrobromate on the oxides of mercury, copper, and lead, on ammonia and on ethylate of sodium were examined.

Corresponding addition products of sulphide of ethyl were also prepared, but owing to the extremely deliquescent character of the hydrobromate of ethyl-thetine, attention was chiefly devoted to the derivatives of the methyl compound.

Iodacetic ether does not form an addition product with sulphide of methyl. The reaction here takes a different direction, free iodine and iodide of trimethylsulphine being produced. The authors are engaged in the investigation of this reaction, and also of the products of the oxidation of the thetine compounds.

## 5. Note on the Various Possible Expressions for the Force Exerted by an Element of one Linear Conductor on an Element of another. By Professor Tait.

In the *Quarterly Mathematical Journal* for 1860, I gave a quaternion process for obtaining in a very simple manner, from Ampère's experimental data, his well-known expression for the mutual action between two elements of currents. As one of the data the assumption was made, after Ampère, that the action is a force whose direction is that of the line joining the middle points of the elements, *i.e.*, it was assumed that the necessary equality of action and reaction holds, not merely for two closed circuits but, for each pair of elements of these circuits. I promised in that paper to publish a more general investigation, in which no such assumption should be made; but I was prevented from doing this by having seen a reference to a memoir by Cellerier, in which it was stated that such an investigation had been given. I did not, till very recently, succeed in getting any information about that memoir, none of which seems indeed to have been printed except a very brief extract in the *Comptes Rendus* for 1850, vol. xxx., giving no details: but the subject was recalled to my memory by Clerk-Maxwell's *Treatise on Electricity, &c.*, in which there is an investi-

gation of the possible expressions for the forces which satisfy Ampère's data without necessarily satisfying his assumption. Both of these authors make the undetermined part of the expression depend upon a single arbitrary function. My investigation leads to two. The question is one of comparatively little physical importance, but I give this investigation for its extreme simplicity.

The following is, as nearly as I can recollect, my original process, which has, at least at first sight, nothing in common with that of Clerk-Maxwell.

1. Ampère's data for closed currents are briefly as follows:—

I. Reversal of either current reverses the mutual effect.

II. The effect of a sinuous or zig-zag current is the same as that of a straight or continuously curved one, from which it nowhere deviates much.

III. No closed current can set in motion a portion of a circular conductor movable about an axis through its centre, and perpendicular to its plane.

IV. In similar systems, traversed by equal currents, the forces are equal.

2. First, let us investigate the expression for the *force* exerted by one element on another.

Let $\alpha$ be the vector joining the elements $\alpha_1$, $\alpha'$, of two circuits; then, by I., II., the action of $\alpha_1$ on $\alpha'$ is *linear* in each of $\alpha_1$, $\alpha'$, and may, therefore, be expressed as

$$\phi\alpha' ,$$

where $\phi$ is a linear and vector function, into each of whose constituents $\alpha_1$ enters linearly.

The resolved part of this along $\alpha'$ is

$$S . U\alpha'\phi\alpha' ,$$

and, by III., this must be a complete differential as regards the circuit of which $\alpha_1$ is an element. Hence,

$$\phi\alpha' = -(S . \alpha_1\nabla)\psi\alpha' + V\alpha'\chi\alpha_1 ,$$

where $\psi$ and $\chi$ are linear and vector functions whose constituents

involve $\alpha$ only. That this is the case follows from the fact that $\phi\alpha'$ is homogeneous and linear in each of $\alpha_1$, $\alpha'$. It farther follows, from IV., that the part of $\phi\alpha'$ which does not disappear after integration round each of the closed circuits is of no dimensions in $T\alpha$, $T\alpha'$, $T\alpha_1$. Hence $\chi$ is of $-2$ dimensions in $T\alpha$, and thus

$$\chi\alpha_1 = \frac{p\alpha S\alpha\alpha_1}{T\alpha^4} + \frac{q\alpha_1}{T\alpha^2} + \frac{rV\alpha\alpha_1}{T\alpha^3}$$

where $p$, $q$, $r$ are numbers.

Hence we have

$$\phi\alpha' = -S(\alpha_1\nabla)\psi\alpha' + \frac{pV\alpha\alpha' S\alpha\alpha_1}{T\alpha^4} + \frac{qV\alpha'\alpha_1}{T\alpha^2} + \frac{rV.\alpha' V\alpha\alpha_1}{T\alpha^3}.$$

Change the sign of $\alpha$ in this, and interchange $\alpha'$ and $\alpha_1$, and we get the action of $\alpha'$ on $\alpha_1$. This, with $\alpha'$ and $\alpha_1$ again interchanged, and the sign of the whole changed, should reproduce the original expression—since the effect depends on the relative, not the absolute, positions of $\alpha$, $\alpha_1$, $\alpha'$. This gives at once,

$$p = 0\ ,\ q = 0,$$

and

$$\phi\alpha' = -S(\alpha_1\nabla)\psi\alpha' + \frac{rV.\alpha' V\alpha\alpha_1}{T\alpha^3},$$

with the condition that the first term changes its sign with $\alpha$, and thus that

$$\psi\alpha' = \alpha S\alpha\alpha' \mathsf{F}(T\alpha) + \alpha' F(T\alpha),$$

which, by change of $\mathsf{F}$, may be written

$$= \alpha S(\alpha'\nabla) f(T\alpha) + \alpha' F(T\alpha),$$

where $f$ and F are any scalar functions whatever.

Hence

$$\phi\alpha' = -S(\alpha_1\nabla)[\alpha S(\alpha'\nabla)f(T\alpha) + \alpha' F(T\alpha)] + \frac{rV.\alpha' V\alpha\alpha_1}{T\alpha^3}.$$

which is the general expression required.

3. The simplest possible form for the action of one current-element on another is, therefore,

$$\phi\alpha' = \frac{rV.\alpha' V\alpha\alpha_1}{T\alpha^3}.$$

Here it is to be observed that Ampère's *directrice* for the circuit $\alpha_1$ is

$$\theta = \int \frac{V\alpha\alpha_1}{T\alpha^3} \,,$$

the integral extending round the circuit; so that, finally,

$$\varphi\alpha' = -\, rS\alpha_1\nabla V\alpha'\theta \,.$$

4. We may obtain from the general expression above the absolutely symmetrical form,

$$\frac{rV.\alpha'\alpha\alpha_1}{T\alpha^3} \,,$$

if we assume

$$f(T\alpha) = \text{const} \,,\; F(T\alpha) = \frac{r}{T\alpha} \,.$$

Here the action of $\alpha'$ on $\alpha_1$ is parallel and equal to that of $\alpha_1$ on $\alpha'$. The forces, in fact, form a couple, for $\alpha$ is to be taken negatively for the second—and their common direction is the vector drawn to the corner $a$ of a spherical triangle $abc$, whose sides $ab$, $bc$, $ca$ in order are bisected by the extremities of the vectors $U\alpha'$, $U\alpha$, $U\alpha_1$. Compare Hamilton's *Lectures on Quaternions*, §§ 223–227.

5. To obtain Ampère's form for the effect of one element on another write, in the general formula above,

$$f(T\alpha) = \frac{r}{T\alpha} \,,\; F(T\alpha) = 0 \,,$$

and we have

$$\begin{aligned}
\frac{1}{r}\varphi\alpha' &= -\, S\alpha_1\nabla\left[ -\frac{\alpha S\alpha\alpha'}{T\alpha^3}\right] + \frac{V.\alpha' V\alpha\alpha_1}{T\alpha^3} \,, \\
&= -\,\frac{\alpha_1 S\alpha\alpha'}{T\alpha^3} - \frac{\alpha S\alpha_1\alpha'}{T\alpha^3} - \frac{3\alpha S\alpha\alpha' S\alpha\alpha_1}{T\alpha^5} + \frac{V.\alpha' V\alpha\alpha_1}{T\alpha^3} \,, \\
&= +\,\frac{2\alpha}{T\alpha^5}\left(\alpha^2 S\alpha_1\alpha' - \frac{3}{2} S\alpha\alpha' S\alpha\alpha_1\right), \\
&= -\,\frac{2\alpha}{T\alpha^5}\left(S.V\alpha\alpha' V\alpha\alpha_1 + \frac{1}{2} S\alpha\alpha' S\alpha\alpha_1\right),
\end{aligned}$$

which are the usual forms.

6. The remainder of the expression, containing the arbitrary terms, is of course still of the form

$$-\, S(\alpha_1\nabla)\left[\alpha S(\alpha'\nabla) f(T\alpha) + \alpha' F(T\alpha)\right].$$

In the ordinary notation this expresses a force whose components are proportional to

(1.) Along $\alpha$ $\qquad -r\dfrac{d^2f}{ds_1 ds'}$,

(Note that, in *this* expression, $r$ is the distance between the elements.)

(2.) Parallel to $\alpha'$ $\qquad \dfrac{dF}{ds_1}$,

(3.) Parallel to $\alpha_1$ $\qquad -\dfrac{df}{ds'}$.

If we assume $f = F = -Q$, we obtain the result given by Clerk-Maxwell (*Electricity and Magnetism*, § 525), which differs from the above only because he assumes that the force exerted by one element on another when the first is parallel and the second perpendicular to the line joining them is *equal* to that exerted when the first is perpendicular and the second parallel to that line.

7. What precedes is, of course, only a particular case of the following interesting problem:—

*Required the most general expression for the mutual action of two rectilinear elements, each of which has dipolar symmetry in the direction of its length, and which may be resolved and compounded according to the usual kinematical law.*

The data involved in this statement are equivalent to I. and II. of Ampère's data above quoted. Hence, keeping the same notation as in § 2 above, the force exerted by $\alpha_1$ on $\alpha'$ must be expressible as

$$\phi\alpha'$$

where $\phi$ is a linear and vector function, whose constituents are linear and homogeneous in $\alpha_1$; and, besides, involve only $\alpha$.

By interchanging $\alpha_1$ and $\alpha'$, and changing the sign of $\alpha$, we get the force exerted by $\alpha'$ on $\alpha_1$. If in this we again interchange $\alpha_1$ and $\alpha'$, and change the sign of the whole, we must obviously reproduce $\phi\alpha'$. Hence we must have $\phi\alpha'$ changing its sign with $\alpha$, or

$$\phi\alpha' = P\alpha S\alpha_1\alpha' + Q\alpha S\alpha\alpha_1 S\alpha\alpha' + R\alpha_1 S\alpha\alpha' + \mathsf{R}\alpha' S\alpha\alpha_1$$

where P, Q, R, $\mathsf{R}$ are functions of $T\alpha$ only.

8. The vector *couple* exerted by $\alpha_1$ on $\alpha'$ must obviously be expressible in the form

$$V.\alpha'\varpi\alpha_1,$$

where $\varpi$ is a new linear and vector function depending on $\alpha$ alone. Hence its most general form is

$$\varpi\alpha_1 = P\alpha_1 + Q\alpha S\alpha\alpha_1,$$

where P and Q are functions of $T\alpha$ only. The form of these functions, whether in the expression for the force or for the couple, depends on the special data for each particular case. Symmetry shows that there is no term such as

$$RV\alpha\alpha_1.$$

9. As an example, let $\alpha_1$ and $\alpha'$ be elements of solenoids or of uniformly and linearly magnetised wires, it is obvious that, as a closed solenoid or ring-magnet exerts no external action,

$$\phi\alpha' = -S\alpha_1\nabla.\psi\alpha'.$$

Thus we have introduced a different datum in place of Ampère's No. III. But in the case of solenoids the Third Law of Newton holds—hence

$$\phi\alpha' = S\alpha_1\nabla S\alpha'\nabla.\chi\alpha,$$

where $\chi$ is a linear and vector function, and can therefore be of no other form than

$$\alpha f(T\alpha).$$

Now two solenoids, each extended to infinity in one direction, act on one another like two magnetic poles, so that (this being our equivalent for Ampère's datum No. IV.)

$$\chi\alpha = p\frac{\alpha}{T\alpha^3}.$$

Hence the vector force exerted by one small magnet on another is

$$pS\alpha_1\nabla S\alpha'\nabla\frac{\alpha}{T\alpha^3}.$$

10. For the couple exerted by one element of a solenoid, or of a uniformly and longitudinally magnetised wire, on another, we have of course the expression

$$V.\alpha'\varpi\alpha_1,$$

where $\varpi$ is some linear and vector function.

Here, in the first place, it is obvious that

$$\varpi\alpha_1 = -S\alpha_1\nabla.\frac{\alpha}{F(T\alpha)};$$

for the couple vanishes for a closed circuit of which $\alpha_1$ is an element, and the integral of $\varpi\alpha_1$ must be a linear and vector function of $\alpha$ alone. It is easy to see that in this case

$$F(T\alpha) \propto (T\alpha)^3.$$

11. If, again, $\alpha_1$ be an element of a solenoid, and $\alpha'$ an element of current, the force is

$$\phi\alpha' = -S\alpha_1\nabla.\psi\alpha',$$

where

$$\psi\alpha' = P\alpha' + Q\alpha S\alpha\alpha' + RV\alpha\alpha'.$$

But no portion of a solenoid can produce a force on an element of current in the direction of the element, so that

$$\phi\alpha' = V.\alpha'\chi\alpha_1,$$

so that

$$P = 0, \quad Q = 0,$$

and we have

$$\phi\alpha' = -S\alpha_1\nabla(RV\alpha\alpha').$$

This must be of $-1$ linear dimensions when we integrate for the effect of one pole of a solenoid, so that

$$R = \frac{p}{T\alpha^3}.$$

If the current be straight and infinite each way, its equation being

$$\alpha = \beta + x\gamma,$$

where

$$T\gamma = 1 \text{ and } S\beta\gamma = 0,$$

we have, for the whole force exerted on it by the pole of a solenoid, the expression

$$p\beta\gamma\int_{-\infty}^{+\infty}\frac{dx}{(T\beta^2+x^2)^{\frac{3}{2}}} = -2p\beta^{-1}\gamma,$$

which agrees with known facts.

12. Similarly, for the couple produced by an element of a solenoid on an element of a current we have

$$V\alpha'\varpi\alpha_1,$$

where

$$\varpi\alpha_1 = - S\alpha_1\nabla . \psi\alpha ,$$

and it is easily seen that

$$\psi\alpha = \frac{r\alpha}{T\alpha^3} .$$

13. In the case first treated, the couple exerted by one current-element on another is (§ 8 above)

$$V . \alpha'\varpi\alpha_1 ,$$

where, of course, $\pm \varpi\alpha_1$ are the vector forces applied at either end of $\alpha'$. Hence the work done when $\alpha'$ changes its direction is

$$- S.\delta\alpha'\varpi\alpha_1 ,$$

with the condition

$$S.\alpha'\delta\alpha' = 0 .$$

So far, therefore, as change of direction of $\alpha'$ alone is concerned, the mutual potential energy of the two elements is of the form

$$S.\alpha'\varpi\alpha_1 .$$

This gives, by the expression for $\varpi$ in § 8, the following value

$$PS\alpha'\alpha_1 + QS\alpha\alpha'S\alpha\alpha_1 .$$

Hence, integrating round the circuit of which $\alpha_1$ is an element, we have (*On Green's and other Allied Theorems*, § 11, *Trans.* R.S.E., 1869–70)

$$\begin{aligned} \int(PS\alpha'\alpha_1 + QS\alpha\alpha'S\alpha\alpha_1) &= \iint ds_1 S.U\nu_1\nabla(P\alpha' + \dot{Q}\alpha S\alpha\alpha') , \\ &= \iint ds_1 S.U\nu_1 \left(\frac{\alpha\alpha'P'}{T\alpha} - \alpha'\alpha Q\right) , \\ &= \iint ds_1 S.U\nu_1 V\alpha\alpha'\Phi , \end{aligned}$$

where

$$\Phi = \frac{P'}{T\alpha} + Q .$$

Integrating this round the other circuit we have for the mutual potential energy of the two, so far as it depends on the expression above, the value

$$\begin{aligned} &\iint ds_1 S.U\nu_1 \int V\alpha\alpha'\Phi \\ &= - \iint ds_1 S.U\nu_1 \iint ds' V.V(U\nu'\nabla)\alpha\Phi \\ &= \iint ds_1 \iint ds' \left\{ S.U\nu_1 U\nu'(2\Phi + T\alpha\Phi') + S\alpha U\nu' S\alpha U\nu_1 \frac{\Phi'}{T\alpha} \right\} . \end{aligned}$$

But, by Ampère's result, that two closed circuits act on one another as two magnetic shells, it should be

$$\iint ds_1 \iint ds' \mathrm{S.U}\nu_1 \nabla \mathrm{S.U}\nu' \nabla \frac{1}{\mathrm{T}\alpha}$$

$$= \iint ds_1 \iint ds' \left( \mathrm{S.U}\nu_1 \mathrm{U}\nu' \frac{1}{\mathrm{T}\alpha^3} + 3\,\mathrm{S}\alpha\mathrm{U}\nu'\mathrm{S}\alpha\mathrm{U}\nu_1 \frac{1}{\mathrm{T}\alpha^5} \right).$$

Comparing, we have

$$\left.\begin{aligned} \frac{1}{\mathrm{T}\alpha^3} &= 2\Phi + \mathrm{T}\alpha\Phi' \\ \frac{3}{\mathrm{T}\alpha^3} &= \mathrm{T}\alpha\Phi' \end{aligned}\right\},$$

giving

$$\Phi = -\frac{1}{\mathrm{T}\alpha^3}, \quad \Phi' = \frac{3}{\mathrm{T}\alpha^4},$$

which are consistent with one another, and which lead to

$$\frac{\mathrm{P}'}{\mathrm{T}\alpha} + \mathrm{Q} = -\frac{1}{\mathrm{T}\alpha^3}.$$

Hence, if we put

$$\mathrm{Q} = \frac{1-n}{2n\mathrm{T}\alpha^3},$$

we get

$$\mathrm{P} = \frac{1+n}{2n\mathrm{T}\alpha},$$

and the mutual potential of two elements is of the form

$$(1+n)\frac{\mathrm{S}\alpha'\alpha_1}{\mathrm{T}\alpha} + (1-n)\frac{\mathrm{S}\alpha\alpha'\mathrm{S}\alpha\alpha_1}{\mathrm{T}\alpha^3},$$

which is the expression employed by Helmholtz in his recent paper. (*Ueber die Bewegungsgleichungen der Electricität, Crelle,* 1870, p. 76.)

*Monday, 22d December* 1873.

Sir W. THOMSON, President, in the Chair.

Professor Andrews, Hon. F.R.S.E., Vice-President of Queen's College, Belfast, gave an Address on Ozone.

*Monday, 5th January* 1874.

Professor Sir WILLIAM THOMSON, President, in the Chair.

The following Communications were read:—

1. A new Method of Determining the Material and Thermal Diffusivities of Fluids. By Sir William Thomson.

2. Continuants—A New Special Class of Determinants. By Thomas Muir, M.A., Assistant to the Professor of Mathematics in the University of Glasgow.

1. A determinant which has the elements lying outside the principal diagonal and the two bordering minor diagonals each equal to zero, and which has the elements of one of these minor diagonals each equal to negative unity, may be called a *Continuant*. Thus

$$\begin{vmatrix} a_1 & b_1 & 0 & 0 \\ -1 & a_2 & b_2 & 0 \\ 0 & -1 & a_3 & b_3 \\ 0 & 0 & -1 & a_4 \end{vmatrix}$$

is a continuant of the fourth order.

2. A continuant is evidently a function of the elements of the principal diagonal and the variable minor diagonal, and of these alone. Let this function be denoted by K. The above continuant, for example, may then be written

$$K\begin{pmatrix} b_1 & b_2 & b_3 & \\ a_1 & a_2 & a_3 & a_4 \end{pmatrix}.$$

3. By the cyclical transposition of rows and thereafter of columns, we establish a first law of continuants, viz.:—

$$K\begin{pmatrix} b_1 \ldots\ldots b_{n-1} \\ a_1 \; a_2 \ldots\ldots a_{n-1} \; a_n \end{pmatrix} = K\begin{pmatrix} b_{n-1} \ldots\ldots b_1 \\ a_n \; a_{n-1} \ldots\ldots a_2 \; a_1 \end{pmatrix} \quad . \quad \text{(I.)}$$

4. By expansion of the continuant in terms of its principal minors we have

$$K\begin{pmatrix} b_1 \ldots\ldots b_{n-1} \\ a_1 a_2 \ldots a_{n-1} a_n \end{pmatrix} = a_1 K\begin{pmatrix} b_2 \ldots\ldots b_{n-1} \\ a_2 a_3 \ldots a_{n-1} a_n \end{pmatrix} + b_1 K\begin{pmatrix} b_3 \ldots b_{n-1} \\ a_3 \ldots a_{n-1} a_n \end{pmatrix} \quad \text{(II.)}$$

5. From this we see how to evaluate a continuant for special values of its elements, and also to change a continuant into the ordinary notation, *i.e.*, to free it of determinant forms. Thus,

$$K\begin{pmatrix} 4 \; 6 \; 8 \; 9 \; 7 \\ 7 \; 2 \; 3 \; 1 \; 4 \; 5 \end{pmatrix}$$

would be evaluated by first evaluating $K\begin{pmatrix} 7 \\ 4 \; 5 \end{pmatrix}$, thence $K\begin{pmatrix} 9 \; 7 \\ 1 \; 4 \; 5 \end{pmatrix}$, thence $K\begin{pmatrix} 8 \; 9 \; 7 \\ 3 \; 1 \; 4 \; 5 \end{pmatrix}$, and so on.

6. By means of Laplace's expansion-theorem we can establish a result which includes (II.) viz.,

$$K\begin{pmatrix} b_1 \ldots\ldots\ldots\ldots b_{n-1} \\ a_1 a_2 \ldots a_p \ldots a_{n-1} a_n \end{pmatrix} = K\begin{pmatrix} b_1 \ldots b_{p-1} \\ a_1 a_2 \ldots\ldots a_p \end{pmatrix} K\begin{pmatrix} b_{p+1} \ldots b_{n-1} \\ a_{p+1} \ldots\ldots a_n \end{pmatrix}$$

$$+ b_p K\begin{pmatrix} b_1 \ldots b_{p-2} \\ a_1 \ldots\ldots a_{p-1} \end{pmatrix} K\begin{pmatrix} b_{p+2} \ldots b_{n-1} \\ a_{p+2} \ldots\ldots a_n \end{pmatrix} \quad \text{(III.)};$$

and, using instead the present author's extension of Laplace's theorem, we arrive at a still more general proposition, viz.,

$$K\begin{pmatrix} b_1 \ldots b_{n-1} \\ a_1 a_2 \ldots\ldots a_n \end{pmatrix} K\begin{pmatrix} b_h \ldots b_{p-1} \\ a_h \ldots\ldots a_p \end{pmatrix}$$

$$= K\begin{pmatrix} b_1 \ldots b_{p-1} \\ a_1 \ldots\ldots a_p \end{pmatrix} K\begin{pmatrix} b_h \ldots b_{n-1} \\ a_h \ldots\ldots a_n \end{pmatrix}$$

$$+ (-1)^{p-h+1} b_{h-1} b_h \ldots b_p K\begin{pmatrix} b_1 \ldots b_{h-3} \\ a_1 \ldots\ldots a_{h-2} \end{pmatrix} K\begin{pmatrix} b_{p+2} \ldots b_{n-1} \\ a_{p+2} \ldots\ldots a_n \end{pmatrix} \quad \text{(IV.)},$$

where of course $h<p<n$. An important particular case is that for which $p=n-1$ and $h=2$.

7. Another result which is easily proved by induction is

$$K\begin{pmatrix} b_1 \ldots\ldots\ldots\ldots b_{n-1} \\ -a_1, -a_2 \ldots -a_{n-1}, -a_n \end{pmatrix} = (-1)^n K\begin{pmatrix} b_1 \ldots\ldots b_{n-1} \\ a_1 a_2 \ldots a_{n-1} a_n \end{pmatrix} \quad . \quad (\text{V.})$$

8. In any continuant

$$K\begin{pmatrix} b_1 \ldots\ldots b_{n-1} \\ a_1 a_2 \ldots a_{n-1}\, a_n \end{pmatrix}$$

we may call $a_1 a_2 \ldots\ldots a_{n-1} a_n$ the *main diagonal*, and $b_1 b_2 \ldots\ldots b_{n-1}$ the *minor diagonal;* $a_1, a_2, \ldots\ldots, b_1, b_2, \ldots$ being known as *elements.* When each element of the minor diagonal is unity, the continuant may be called *simple*, and in writing such continuants we may agree to omit the minor diagonal, putting, for example,

$$K(a_1 a_2 a_3 \ldots\ldots a_{n-1} a_n) \text{ for } K\begin{pmatrix} 1\ 1 \ldots\ldots 1 \\ a_1\, a_2\, a_3 \ldots a_{n-1} a_n \end{pmatrix}.$$

9. If the elements of the first column of the determinant $K(1\ a_1 a_2 \ldots a_n)$ be subtracted from the corresponding elements of the second column, it will be seen that

$$K(1, a_1, a_2, \ldots a_n) = K(a_1 + 1, a_2, \ldots a_n) \quad . \quad (\text{VI.})$$

10. From (II.) it is clear that

$$K(0, a_2, a_3, \ldots a_n) = K(a_3 \ldots a_n),$$

thence, with the help of (III.), we can show that

$$K(\ldots a, b, c, 0, e, f, g, \ldots) = K(\ldots a, b, c+e, f, g, \ldots) \quad (\text{VII.}),$$

and from this that

$$K(\ldots a, b, c, 0, 0, 0, e, f, \ldots) = K(\ldots a, b, c+e, f, \ldots)$$

and so, generally, when the number of consecutive zero elements is odd.

11. Similarly, from (II.)

$$K(0, 0, a_3, a_4, \ldots a_n) = K(a_3, a_4, \ldots a_n),$$

and from this, with the help of (III.), we can prove that

$$K(\ldots a, b, 0, 0, e, f, \ldots) = K(\ldots a, b, e, f, \ldots) \quad . \quad \text{(VIII.)},$$

and so, generally, when the number of consecutive zero elements is even.

12. Using the ordinary process of finding the greatest common measure of two numbers, we may establish another special property of simple continuants, viz., that, whatever $a_1, a_2, \ldots$ may be,

$$K(a_1, a_2, \ldots a_{n-1}, a_n)$$

is prime to

$$K(a_1, a_2, \ldots a_{n-1}), \; K(a_2, \ldots a_{n-1}, a_n), \; K(a_1 - 1, a_2, \ldots a_n), \text{ and } K(a_1, a_2, \ldots a_n - 1).$$

13. When both diagonals of a continuant are the same when read backwards as when read forwards, it may be called *symmetrical.*

In connection with simple symmetrical continuants, the following identities may be mentioned:—

$$K(a_1, a_2, \ldots a_{n-1}, a_n, a_{n-1}, \ldots a_2, a_1) = K(a_1, a_2, \ldots a_{n-1})\{K(a_1, a_2, \ldots a_{n-2}) + K(a_1, a_2, \ldots a_n)\} \qquad \text{(IX.)}$$

$$K(a_1, a_2, \ldots a_n, a_n, \ldots a_2, a_1) = K(a_1, a_2, \ldots a_{n-1})^2 + K(a_1, a_2, \ldots a_n)^2 \qquad \text{(X.)}$$

$$a_n K(a_1, a_2, \ldots a_{n-1}, a_n, a_{n-1}, \ldots a_2, a_1) = K(a_1, a_2 \ldots a_n)^2 - K(a_1, a_2, \ldots a_{n-2})^2 \qquad \text{(XI.)}$$

$$K(a_1, a_2, \ldots a_{n-1}, 2a_n, a_{n-1} \ldots a_2, a_1) = 2K(a_1, a_2, \ldots a_{n-1})\, K(a_1, a_2, \ldots a_n) \qquad \text{(XII.)}$$

## *Connection between Continuants and Continued Fractions.*

14. The value of the special study of this class of determinants lies in the fact that by means of them the convergents of a con-

tinued fraction are expressible in an unexpectedly simple and elegant manner. Thus—

$$x + \frac{y}{z} = \frac{xz + y}{z} = \frac{\mathrm{K}\begin{pmatrix} y \\ x\,,\,z \end{pmatrix}}{\mathrm{K}(z)};$$

$$v + \frac{w}{x +}\,\frac{y}{z} = v + \cfrac{w}{\cfrac{\mathrm{K}\begin{pmatrix} y \\ x\,,\,z \end{pmatrix}}{\mathrm{K}(z)}}$$

$$= \frac{v\mathrm{K}\begin{pmatrix} y \\ x\,,\,z \end{pmatrix} + w\mathrm{K}(z)}{\mathrm{K}\begin{pmatrix} y \\ x\,,\,z \end{pmatrix}}$$

$$= \frac{\mathrm{K}\begin{pmatrix} w \quad y \\ v\,,\,x\,,\,z \end{pmatrix}}{\mathrm{K}\begin{pmatrix} y \\ x\,,\,z \end{pmatrix}};$$

and thus by induction we prove that—

$$a_1 + \frac{b_1}{a_2 +}\,\frac{b_2}{a_3 +} \dots + \frac{b_{n-2}}{a_{n-1} +}\,\frac{b_{n-1}}{a_n} = \frac{\mathrm{K}\begin{pmatrix} b_1 \quad b_2 \dots\dots b_{n-1} \\ a_1\,,\,a_2\,,\,a_3 \dots a_{n-1}\,,\,a_n \end{pmatrix}}{\mathrm{K}\begin{pmatrix} b_2 \dots\dots b_{n-1} \\ a_2\,,\,a_3 \dots a_{n-1}\,,\,a_n \end{pmatrix}} \quad . \; . \quad \text{(XIII.)}$$

15. In virtue of this connection continuants will be found of the utmost aid in investigating the properties of continued fractions. The following are a few instances of this relating to those continued fractions which are expressible in the form of quadratic surds.

16. Consider the periodic continued fraction—

$$\mathrm{A} + \frac{b_1}{\underset{*}{a_1} +}\,\frac{b_2}{a_2 +}\,\frac{b_3}{a_3 +} \dots\dots + \frac{b_3}{a_2 +}\,\frac{b_2}{a_1 +}\,\frac{b_1}{\underset{*}{2\mathrm{A}} +} \dots$$

where the asterisks are used like the superposed dots in the notation of decimal fractions to indicate the recurring portion or period.

Denoting it by $x$, we have

$$x = \mathrm{A} + \frac{b_1}{a_1 +}\,\frac{b_2}{a_2 +} \cdots + \frac{b_3}{a_2 +}\,\frac{b_2}{a_1 +}\,\frac{b}{2\mathrm{A} + x - \mathrm{A}}$$

$$= \frac{\mathrm{K}\left(\begin{matrix} & b_1 & b_2 \ldots\ldots & b_2 & b_1 & \\ \mathrm{A}, & a_1, & a_2 \ldots\ldots & a_2, & a_1, & \mathrm{A}+x \end{matrix}\right)}{\mathrm{K}\left(\begin{matrix} & b_2 \ldots\ldots & b_2 & b_1 & \\ a_1, & a_2 \ldots\ldots & a_2, & a_1, & \mathrm{A}+x \end{matrix}\right)},$$

whence it can be shown that

$$x^2\mathrm{K}\left(\begin{matrix} & b_2 \ldots\ldots & b_2 & \\ a_1, & a_2 \ldots\ldots & a_2, & a_1 \end{matrix}\right) = \mathrm{K}\left(\begin{matrix} & b_1 & b_2 \ldots\ldots & b_2 & b_1 & \\ \mathrm{A}, & a_1, & a_2 \ldots\ldots & a_2, & a_1, & \mathrm{A} \end{matrix}\right)$$

and thus we have the theorem—

$$\mathrm{A} + \frac{b_1}{\underset{*}{a_1} +}\,\frac{b_2}{a_2 +}\,\frac{b_3}{a_3 +} \cdots + \frac{b_3}{a_2 +}\,\frac{b_2}{a_1 +}\,\frac{b_1}{\underset{*}{2\mathrm{A}} +} \cdots$$

$$= \sqrt{\frac{\mathrm{K}\left(\begin{matrix} & b_1 & b_2 \ldots\ldots & b_2 & b_1 & \\ \mathrm{A}, & a_1, & a_2 \ldots\ldots & a_2, & a_1, & \mathrm{A} \end{matrix}\right)}{\mathrm{K}\left(\begin{matrix} & b_2 \ldots\ldots & b_2 & \\ a_1, & a_2 \ldots\ldots & a_2, & a_1 \end{matrix}\right)}} \quad . \quad . \quad \text{(XIV.)}$$

17. From (XIV.) it is easy to deduce a series of identities expressed in continuants, viz.,

$$\frac{\mathrm{K}\left(\begin{matrix} & b_1 & b_2 \ldots\ldots & b_2 & b_1 & \\ \mathrm{A}, & a_1, & a_2 \ldots & a_2, & a_1, & \mathrm{A} \end{matrix}\right)}{\mathrm{K}\left(\begin{matrix} & b_2 \ldots\ldots & b_2 & \\ a_1, & a_2 \ldots & a_2, & a_1 \end{matrix}\right)} = \frac{\mathrm{K}\left(\begin{matrix} & b_1 & b_2 \ldots\ldots & b_2 & [b_1 & & b_1 & b_2 \ldots\ldots & b_2 & b_1 & \\ \mathrm{A}, & a_1, & a_2 \ldots & a_2, & a_1, & 2\mathrm{A}, & a_1, & a_2 \ldots & a_2, & a_1, & \mathrm{A} \end{matrix}\right)}{\mathrm{K}\left(\begin{matrix} & b_2 \ldots\ldots & b_2 & b_1 & & b_1 & b_2 \ldots\ldots & b_2 & \\ a_1, & a_2 \ldots & a_2, & a_1, & 2\mathrm{A}, & a_1, & a_2 \ldots & a_2, & a_1 \end{matrix}\right)}, \text{ \&c. (XV.)}$$

18. With the help of (XIV.) we can also establish an important proposition in reference to the well-known subject of the expression of the square root of an integer as a continued fraction with unit-numerators. The proposition is:—The general expression for every integer whose square root when expressed as a continued fraction with unit numerators has $q_1, q_2, \ldots q_2, q_1$ for the symmetric portion of its cycle of partial denominators is

$$\left\{\tfrac{1}{2}\mathrm{K}(q_1, q_2, \ldots q_2, q_1)\,m - (-1)^l\,\tfrac{1}{2}\mathrm{K}(q_1, q_2, \ldots q_2)\,\mathrm{K}(q_2 \ldots q_2)\right\}^2$$
$$+ \mathrm{K}(q_1 \ldots q_2)\,m - (-1)^l\,\mathrm{K}(q_2 \ldots q_1)^2 \quad . \quad \text{(XVI.)},$$

$l$ being the number of elements in the cycle.

This is established by taking the general expression for *every* such number, fractional as well as integral, viz.,

$$\frac{K(A, q_1, q_2 \dots q_2, q_1, A)}{K(q_1, q_2 \dots q_2, q_1)} \quad . \quad . \qquad (a),$$

and proceeding to determine what form for A is necessary and sufficient to make this expression integral. The form found is

$$\tfrac{1}{2} K(q_1 \dots q_1) m - (-1)^l \tfrac{1}{2} K(q_1 \dots q_2) K(q_2 \dots q_2),$$

and substituting this for A in $(a)$, we arrive at the expression (XVI.) after some reduction.

19. Further, no integer can be found whose square root when expressed as a continued fraction with unit-numerators has $q_1, q_2 \dots q_2, q_1$ for the symmetric portion of its cycle of partial denominators, unless either $K(q_1 \dots q_2)$ or $K(q_2 \dots q_2)$ be even. This is deducible from the preceding.

20. Many interesting results may also be arrived at in reference to the possibility of expressing in more ways than one by a continued fraction the square root of any number.

All that is requisite in order to find as an equivalent for any quadratic surd, $\sqrt{13}$ say, a periodic continued fraction with a period of any given number of elements, say 5, is the solution in integers of an indeterminate equation of the form

$$\frac{K\begin{pmatrix} & b_1 & b_2 & b_3 & b_2 & b_1 & \\ A, & a_1, & a_2, & a_2, & a_1, & A \end{pmatrix}}{K\begin{pmatrix} & b_2 & b_3 & b_2 & \\ a_1, & a_2, & a_2, & a_1 \end{pmatrix}} = 13.$$

21. This leads to the consideration of the various identical forms of periodic continued fractions, and on this subject much may be learned. As an instance, we may show how a continued fraction with unit-numerators, such as is found in the usual way as the equivalent of a quadratic surd, may always be reduced to a periodic continued fraction with only three elements in its period. The identity is

$$A + \frac{1}{\underset{*}{a}+}\frac{1}{b+}\dots\frac{1}{b+}\frac{1}{a+}\frac{1}{\underset{*}{2A}+}\dots$$

$$= A + \frac{K(b\,c \dots c\,b)}{\underset{*}{K(a\,b \dots c\,b)}+}\frac{(-1)^{l-1}}{K(a\,b \dots c\,b)+}\frac{K(b\,c \dots c\,b)}{\underset{*}{2A}}+\dots$$

where $l$ is the number of elements in the period of the first fraction. This we may prove by deducing from the expression which is given by (XIV.) for the square of the right hand member, the expression also given by (XIV.) for the square of the left hand member.

Similarly, we may show that

$$A + \frac{1}{\overset{*}{a}+} \frac{1}{b+} \frac{1}{c+} \frac{1}{b+} \frac{1}{a+} \frac{1}{\overset{*}{2A}+} \cdots$$

$$= A + \frac{bc+2}{\overset{*}{abc+2a+c}+} \frac{1}{b+} \frac{1}{abc+2a+c+} \frac{bc+2}{\overset{*}{2A}} + \cdots$$

and many other such identities.

22. Lastly, it is easily demonstrated that the condition that any periodic continued fraction

$$A + \frac{a_1}{\overset{*}{b_1}+} \frac{a_2}{b_2+} \cdots + \frac{a_{n-1}}{b_{n-1}+} \frac{a_n}{\overset{*}{b_n}+} \cdots$$

may represent a quadratic surd is

$$K\begin{pmatrix} a_1 \ldots\ldots a_{n-1} \\ A, b_1 \ldots b_{n-2}, b_{n-1} \end{pmatrix} = K\begin{pmatrix} a_2 \ldots\ldots\ldots a_n \\ b_1, b_2 \ldots b_{n-1}, b_n - A \end{pmatrix},$$

and that this can be satisfied in other ways than by choosing the elements so that the diagonals of the one continuant when read forward may be the same as those of the other when read backward.

## 3. Remarks upon the Footprints of the Dinornis in the Sand Rock at Poverty Bay, New Zealand, and upon its recent extinction. By T. H. Cockburn-Hood, F.G.S.

Impressions of the tracks of large birds from this locality have lately been objects of attraction to visitors to the museum at Wellington, New Zealand. To these Dr Hector, F.R.S., has affixed a label, stating that they are from the "Sea shore sand" at Poverty bay, a harbour on the east coast of the north island. "Sand rock" would have been a preferable term, as to most observers the description is calculated to convey the idea that these footprints are but of yesterday's date. Indeed, were it not probable that the moa was

extinct in the northern island for a considerable time before it was exterminated on the opposite side of Cook's Straits (which is a matter still quite open to doubt), they might be merely the tracks of individuals, contemporary with that, the egg of which was found in the grave of the Hurunui chieftain, placed there to serve him as provision on his way to happier hunting grounds, and would thus lose much of the interest which appertains to them as very ancient memorials.

The present specimens were obtained by the writer on a late visit to the district of Poverty Bay.

The slabs were cut out of a bed of rock, crossing a small affluent which falls into the Turanganui river, near its mouth, and the footprints, first observed by the ferryman, and pointed out to Archdeacon Williams, are now washed by every tide. The deposit can be traced across the estuary to a point under the high land, on the northern shore of the bay, where similar impressions are to be seen.

It has been suggested that this bed is but a portion left of the ancient plateau composed of strata known to local geologists as the Hawke's Bay series, but no such antiquity can be assigned to it, having been formed from the detritus of the cliffs (which rival in whiteness the chalk walls of the English channel) swept into this spot by a current which eddied round under the precipitous coast, at a time when the shallow bay extended further inland, but when otherwise the configuration of the land was much the same as it is now.

From the number of the footprints crossing and recrossing each other, and the proximity of those of individuals, it seems that these birds were in the habit of resorting to the sea-shore to feed upon the small fish and mollusks left by the receding tide, as the Rheas of South America do at the present day.

The strata among which the impressions occur appear to be the result partly of the accumulation of blown sand, partly of subaqueous deposit during a period of gradual submergence.

At the mouth of the Hutt River, and along the shore of Wellington harbour, during the earthquakes of 1855, the land rose nine feet, and a corresponding depression took place of the valley, it is stated, in which the town of Blenheim is situated on the southern shore of Cook's Straits.

At one time this ornithichnite bed, now washed by every tide, was (as it is still beyond its influence) covered by many feet of the delta alluvium. The river Waipaoa, which formed these extensive plains of rich soil, averaging twenty to twenty-five feet in depth, now very rarely overflows its banks. Only once, in the memory of the oldest native, has it done so to any extent, and this was since the settlement of Europeans, on which occasion there was a deposit left of half an inch, in some few spots of an inch of silt; although in bygone times, under different cosmical influences, it probably discharged a much greater volume of water into the bay, at a point opposite the island on the northern shore, and left after every fresh a larger amount of soil than it does now on these rare occasions, a vast time must have elapsed since it left the first layer of mud over the sandstone bed.

Dr. Hochstetter, the accomplished naturalist who accompanied the Austrian expedition of 1859, remarks, "These gigantic birds belong to an era prior to the human race, to a Post-Tertiary period; and it is a remarkably incomprehensible fact of the creation, that whilst at the very same period in the old world, elephants, rhinoceroses, hippopotami—in South America, gigantic sloths and armadillos—in Australia, gigantic kangaroos, wombats, and dasyures were living,—the colossal forms of life were represented in New Zealand by gigantic birds." But whilst these gigantic birds have a higher antiquity than even the megatherium, the diprotodon, or zygomaturus, and other strange quadrupedal forms of life, which have long passed away, or left only puny representatives, like the æpiornis of Madagascar, which maintained its ground down to a late period in that great island, and against men, too, singularly, of an allied race to the Maorie, the moa has the credit of having held its own down to the present century, through all the great changes of scene and climate which have taken place since its ancestors stalked over the plains of the southern portion of a great land,—the backbone of which, and little more, remains,—perhaps with large lacertians for its companions, long after the giant marsupials, the contemporaries of its congener * on the Australian savannahs, had disappeared.

* The interesting discovery there of a large fossil bird has lately been made known by the distinguished geologist the Rev. W. B. Clarke, who first made

The evidences of the late existence of the moa are to be seen. It is not possible that the tender skull and small bones could have been preserved in the situations in which they have been found for any great lapse of time. Exposed to the fierce summer sun, and the severe winter frosts on the upper Otago plains, the bones of a bullock soon decay, but upon these downs nearly perfect skeletons of moas have been found amongst the high fern, with a heap of the so-called moa stones beside them, evidently undisturbed since the birds died upon the spot. The feathers in the museum at Wellington are some of those preserved by the chiefs in the carved boxes which most persons of distinction possessed for the purpose of keeping such prized ornaments. These, and the egg with the well-developed bones of the embryo chick, of which a photograph is here presented,—the extremely interesting relic, the cervical vertebræ of a moa, to which the skin, partially covered with feathers, is still attached by the shrivelled muscles and integuments, found in a cave in Otago formed by an overhanging cliff of mica schist,—are amongst the objects in that collection affording proofs almost incontrovertible, to say nothing of the traditions of the mode of hunting the grand quarry,* preserved in Maorie song and story.

The remains of these gigantic birds are common throughout both islands. No Maorie, upon being shown any of the principal bones, will hesitate in referring them at once to the moa. If credence is denied to their traditions, we are obliged to come to the conclusion that the different tribes possessed persons endowed with the acumen of a Cuvier or an Owen, who explained from

public the marvellous auriferous and general mineral wealth of that continent, and by his indefatigable researches has added so much to our knowledge of its strange denizens in the past, as well as at the present time. Professor Owen has named this bird dromornis, considering it to have been more allied to the emeu than the moa or apterix tribe.

* The paper was accompanied by two photographs. Of these, one was that of the skeleton of one of the largest specimens hitherto obtained. It is placed in the museum at Christ Church, New Zealand, beside that of a tall man. It was one of a great number dug up by Mr Moore at Glenmark in Canterbury province, in a piece of swampy ground, now transformed into a fine garden, which had been one of those places into which the bones of different individuals were washed from the hills around during freshes, and into which the moas rushed when driven by the fires kindled by the natives for the purpose of driving their game, dray loads of bones being here collected.

their knowledge of comparative anatomy that these huge remains appertained to birds. If the theory of the extinction of the dinornis before the arrival of the Maories be accepted, a very great age must be granted to these singularly well-preserved bones; for, from some of the traditions of those people, we are led to the conclusion that the date of their forefathers' landing in this country is much more remote than generally supposed.

It may be that, as well as possessing a knowledge of comparative anatomy, the Maorie fathers were also acute geologists; but it is much more probable that the poetical story of the quarrel of the three brother gods of the volcanos of Rua-pehu, Tonjoriro, and Taranaki, and the flight of the latter down to the plain which now bears his name, tearing up, as he fled, the deep gorge of the Whanjarioa river, the taking of the remarkable truncated cone of Ranjitolo* from the lake on the north shore of Auckland harbour, and other similar stories, have reference to memories of those great disturbances, when the almost matchless cone of Mount-Egmont was thrown up on the Taranaki shore, and the geyser circled lake of Taupo was formed, where the third great crater of the group formerly stood upon "that huge flat cone,"—the sterile pumice-stone plateau of Taupo,—events which took place at a period when the stepping-stones from New Zealand to the old home of the Maorie were probably not so far apart as they are to-day, as far back, it may be, as the time when the skeletons of men of this most ancient type, now from time to time exhumed from their graves deep in the solid limestone rock, covered with the ashes and scoriæ of long quiescent craters, lay bleaching upon the coral strand of Oahu.

* This volcano has evidently been quiet for a long period, but its name, "bloody heavens," denotes that it has not always been so, since the Maories first sailed up Hauraki Gulf.

*Monday, 19th January* 1874.

Principal Sir ALEX. GRANT, Vice-President, in the Chair.

The following Communications were read :—

1. Supplementary Notice of the Fossil Trees of Craigleith Quarry. By Sir R. Christison, Bart., Hon. Vice-President, R.S.E., &c.

*This notice supplements that of 5th May last, which has been published in the Abstracts of the Proceedings of the Society.*

Seven fossils, all apparently belonging to the Pine tribe, and either to the same species, or to two closely allied to one another, have been uncovered since 1826 in the sandstone of Craigleith Quarry. Six are stems of great trees; and one is a longitudinally split section of a large branch, or possibly of another stem. Portions of all seven have been traced as still in existence, and have been subjected more or less to examination. Of one, the greatest of all, about 36 continuous feet, from 12 to 14 feet in girth, have been removed in large fragments to the British Museum, and will be pieced and erected there. Another, found in 1830, is now partly in the Botanic Garden, and will be supplemented by other portions at present in the Museum of Science and Art, so as to make a nearly perfect fossil stem 30 feet in length. A third, nearly 9 feet in girth, has been sliced and polished, to show its structure on the great scale, and will be exhibited in the British Museum, the Edinburgh Museum, and the Edinburgh Botanic Garden.

The composition of all these great fossils is substantially the same. The great mass of each consists of carbonate of lime, carbonate of magnesia, carbonate of protoxide of iron, and free carbon, the proportions varying in different parts of the same fossil. The iron-carbonate and charcoal vary most in their amount. The charcoal, which is left after the action of diluted acids, sometimes without any other insoluble residuum, seems to form three per cent. of the mass, unless when collected, as it often is, in

cavities. This charcoal contains only about 3½ per cent. of incombustible ash.

The surface of the fossils is covered with a shining coat of very bituminous caking coal, which on the principal part of the stem varied from only a 20th to a 10th in thickness, but at the lower end of that now at the British Museum, increased to half an inch, and at last to two inches and a half. This coaly covering contains only 4, 3, 2, and sometimes only 1·1 per cent. of mineral matter; which is not the same as the fossilising matter of the included wood, but is chiefly siliceous in nature, being at least insoluble in acids. The crust is not altered bark, for bark could not fail to undergo, in part at least, fossilisation by the material which has fossilised the wood. Moreover, the coaly crust is found round fragments and on broken points where bark could never have existed.

The rock of the quarry is a very pure quartzy sandstone, hard, tough, and quite free from earthy carbonates or iron. But for some feet around the fossils, and also here and there throughout the quarry, where there is no fossil near, the rock has quite a different appearance, has a higher density, is more sharp-edged, much tougher, and harder to pulverise, and becomes yellow under exposure to the air. These changes are owing to the siliceous particles of the sandstone being bound together by carbonate of lime, carbonate of magnesia, and carbonate of protoxide of iron, forming together from 10 to 38 per cent. of the rock, and bearing much the same relation in proportion to each other as in the mineral material of the fossils,—consequently derived from the same fluid which fossilised them.

Thus the interesting fact is presented of these great trees and the rock in which they are imbedded having been both similarly mineralised, so to speak, by the same fossilising fluid, while there is between them a thin uniform coating of bituminous coal, which has refused admission to any of the fossilising agents. After rejecting various theories to account for this exemption, the only one which stands the test of facts is, that a part of the process of fossilisation consists in a slow process, analogous in its results to the destructive distillation of wood, the result of which is charcoal left behind, and bitumen gradually forced outwards, and collected on the exterior surface.

The charcoal which remains in the stems renders their minute internal structure singularly distinct when a thin transparent slice is placed before the microscope. Longitudinal woody bundles, transverse medullary rays, crowded cells of the longitudinal fibres cut crosswise, are all seen most characteristically; and in one specimen two inches in breadth the boundaries and whole structure of five annual layers of wood are displayed characteristically, even to the naked eye. On the polished surface of one of the great stems, too, the eye can easily trace many annual rings for long distances.

## 2. On a Method of Demonstrating the Relations of the Convolutions of the Brain to the Surface of the Head. By Professor Turner.

The outer surface of the skull does not correspond in shape to the outside of the brain. If it had corresponded there would have been no difficulty in determining the form of the brain from an inspection of the form of the head.

The shape of the brain does correspond to the wall of the cranial cavity. This wall is formed by the inner table of the cranial bones, which table, though separated from the brain itself by the cerebral membranes, is moulded upon the exterior of the organ.

The difference between the form of the inner table of the skull and that of the outside of the cranium is owing to the superaddition of the diplöe and of the outer table, which superadded parts modify the shape of the outer surface of the skull.

The diplöe varies somewhat in thickness in different bones, or in different parts of the same bone, and even at different periods of life, and these variations necessarily cause the outer table to be removed to a greater distance from the inner table in some parts of the cranial wall than in others.

The outer table is modified in shape by ridges and processes for the attachment of muscles; *e.g.*, temporal ridge, curved lines of occiput, occipital protuberance, mastoid process; but in certain localities, as the superciliary ridges, glabella and mastoid processes, more especially in the male skull, it is still further modified by the hollowing out of the diplöe into the frontal and mastoid air cells

or sinuses, and the elevation of the corresponding part of the outer table.

These difficulties in the way of estimating the exact shape of the exterior of the brain, from an inspection of the outside of the head, were pointed out and discussed at the time when the phrenological systems of Gall and Spurzheim were advocated in this city by George Combe and his disciples.

But at that period an additional and even more important difficulty stood in the way of determining the exact relations of the outside of the brain to the outside of the skull, for the external configuration of the brain itself was not properly understood.

Spurzheim had undoubtedly recognised that, in general form and direction, the convolutions of the human brain are "remarkably regular." Thus he says—" The transverse convolutions of the superior, lateral, and middle parts of the hemispheres are never found running in any other direction—never longitudinally, for example. Those that lie longitudinally again, as they do under the squamous suture, behind the temporal bone and on either side of the olfactory nerve, are never met with disposed transversely."* His contemporaries Reil, Rolando, Foville, and Huschke had also directed attention to the constancy of individual convolutions. It was not, however, until the publication in 1854 of Gratiolet's great work on the cerebral convolutions† that the surface of the cerebrum was so mapped out that definite descriptive names were applied, not only to the several lobes, but to the individual convolutions composing them, and the constancy of their position and relations to each other precisely determined. The study of Gratiolet's work, and the adoption by so many anatomists of the greater number of his descriptive terms, have tended materially to advance our knowledge of the convolutions, and to make them more definite objects of physiological and pathological research. A need has therefore arisen for localising the position of the cerebral lobes and convolutions on the surface of the skull and head, and a method, or methods, of readily doing so is to be desiderated. In selecting names for four of the five lobes into which he subdivided each cerebral hemisphere, Gratiolet employed terms which expressed

* The Anatomy of the Brain, translated by Willis, p. 111. London, 1826.
† Mémoires sur les Plis Cerebraux. Paris, 1854.

the relations he believed to exist between these lobes and the vault of the skull, *e.g.* frontal, parietal, occipital, and temporo-sphenoidal lobes. In an essay published in 1861,* M. Broca pointed out that the frontal bone was not equal in extent to the frontal lobe, but that the fissure of Rolando was invariably some distance behind the coronal suture. In eleven males examined the minimum distance of the upper end of this fissure was 40 mm., the maximum 63 mm. from the suture. He further stated that a constant relation existed between the lambdoidal suture and the fissure which separates the parietal from the occipital lobe. He never found the suture more than 15 mm. from the fissure, rarely more than 5 mm. M. Broca's method of determining these relations was by drilling holes in the skull, inserting wooden pegs into the brain, and then, after removing the skull cap, ascertaining the part of the surface of the hemisphere into which the pegs had penetrated. Almost similar results were obtained by Professor Bischoff by pursuing the same mode of examination.†

This plan of drilling holes through the skull, and inserting pegs through them into the brain, is one which may be conveniently employed when the object is merely to obtain an idea of the extent of the lobes of the cerebrum in relation to the surface of the head, as only a few holes require to be bored to effect this object. But as the operation of drilling a number of holes through the cranial bones demands the expenditure of much time and labour, it is not very convenient if it is desired to fix the position of the individual convolutions. It occurred to me, therefore, that some other method might be resorted to to effect this object.

As a preliminary measure, I sub-divided the surface of the skull into regions: a præ-coronal or frontal, the region of the frontal bone; a parietal, sub-divided into antero- and postero-parietal by a vertical line drawn upwards from the squamous suture through the parietal eminence to the sagittal suture; a post-lambdoidal or occipital, between the lambdoidal suture and the superior curved line of the occiput; a squamosal and a sphenoid, corresponding to the squamous temporal and to the great wing of the sphenoid. The line of the temporal ridge sub-divides the antero- and postero-

* Sur le Siége de la Faculté du Langage articulé. Paris, 1861.

† Die Grosshirnwindungen des Menschen. Munich, 1868.

parietal into a supero- and infero-anterior and supero- and infero-posterior parietal regions, and marks off also an infero-frontal area on the frontal bone. The frontal bone may be still further subdivided into a supero-and mid-frontal region by a longitudinal line drawn back from the upper border of the orbit through the frontal eminence to the coronal suture.

With a fine saw I then cut out, one after another, the pieces of bone along the lines which constituted the boundaries of these different regions, and examined with care the particular convolution, or group of convolutions, which lay immediately subjacent to the portion of bone removed. In this manner I was able to localise in the specimens examined the relations of the convolutions to the surface of the skull and head. As I have already detailed the results of my examinations in the "Journal of Anatomy and Physiology," November 1873, I need not repeat them here; but it may not be out of place to point out that the lobes of the brain by no means precisely correspond to the areas of the cranial bones, after which four of them are named. The frontal lobe is not only covered over by the frontal bone, but extends backwards for a considerable distance under cover of the parietal bone. If we accept, as I have elsewhere described,* the fissure of Rolando as the posterior limit of this lobe, then the larger part of the antero-parietal region corresponds with the frontal lobe, for not only does it contain the origins of the superior, middle, and inferior frontal gyri, but also the ascending frontal convolution. But even if we were to regard the ascending frontal gyrus, and not the fissure of Rolando, as bounding the frontal lobe posteriorly, the frontal lobe would still not be wholly localised under cover of the frontal bone, for the superior, middle and inferior frontal gyri all arise from the ascending frontal gyrus, behind the line of the coronal suture.

The occipital lobe also is not limited to the region covered by the squamous part of the occipital bone, but slightly overlapping the lambdoidal suture, extends forwards for a short distance into the back part of the upper postero-parietal area, and through the superior annectent gyrus reaches the parieto-occipital fissure.

* Edinburgh Medical Journal, June 1866, and separate publication, "The Convolutions of the Human Cerebrum topographically considered."

The superior temporo-sphenoidal gyrus, though for the most part situated under cover of the squamous-temporal and great wing of the sphenoid, yet ascends into both the lower antero- and lower postero-parietal areas.

The area covered by the parietal bone, so far then from being conterminous with the parietal lobe of the cerebrum, is trenched on anteriorly, posteriorly and inferiorly by three of the other lobes of the brain. The convolutions of the parietal lobe itself are especially grouped round the parietal eminence, and in the interval between that structure and the sagittal suture.

The Insula or central lobe does not come to the surface, but lies deep in the Sylvian fissure, and is concealed by the convolutions which form the margin of that fissure anteriorly. It lies opposite the upper part of the great wing of the sphenoid and its line of articulation with the antero-inferior angle of the parietal and the squamous part of the temporal.

## 3. On some Peculiarities in the Embryogeny of *Tropæolum speciosum,* Endl. & Poepp., and *T. peregrinum,* L. By Professor Alexander Dickson.

## 4. Notes on Mr Sang's Communication of 7th April 1873 on a Singular Property possessed by the Fluid enclosed in Crystal Cavities in Iceland Spar. (1.) By Professor Tait; (2.) By Professor Swan.

### (1.) Professor Tait.

The very beautiful experiment of Mr Sang, communicated to the Society on the 7th April, 1873, suggested to me, as soon as I heard him read his description of it, an explanation which was confirmed by a subsequent examination of his specimens. Some remarks made to me by members of the Council of the Society, three days afterwards, led me to write, and deposit (under seal, as Mr Sang had announced that he was still prosecuting his inquiry) with the Secretary the following hastily written docu-

ment, which has been since that time in his possession, and is now printed *verbatim* :—

"*April* 10*th*, 1873.

"Carbonic Acid—partly liquid, partly gaseous—fills the cavity.

"Distillation, when one end is heated ever so slightly above the other, the circumstances being of almost unexampled favourability for such an effect. Hence the *apparent* motion of the bubble. IT IS NOT THE SAME BUBBLE AS IT MOVES.

"General problem suggested by this, and easily solved by the dynamical theory of heat.

"Find distribution of LEAST ENTROPY of contents of a vessel where the temperature is a given function of the position in space, and the contents are one or more substances (say, for simplicity, not chemically acting on one another) in two or more different states (as to latent heat, &c.)

"This is *more* (MUCH MORE) than the whole affair.

P. G. TAIT."

A day or two afterwards I tried the experiment on a large scale, with the assistance of my laboratory students, and at once succeeded in showing to them, and to several of my colleagues, Mr Sang's results in quill tubes of three or four inches in length, containing sulphurous acid partly in the liquid and partly in the gaseous state.

The present communication, like that of Professor Swan which follows it, is now made to the Society at the request of Mr Sang himself.

(2.) Professor Swan.

The following note is a narrative of experiments made by me nine months ago, on the 5th and 6th May 1873, on the motions observed in the cavities of Iceland spar by Mr Sang, with an explanation of the manner in which I believe these singular movements to be caused by heat. Being unwilling to interfere with Mr Sang's investigations then in progress, I did not at the time seek to publish my note, but forwarded it in a sealed envelope to the secretary of the Society, in whose custody it has since remained. It is now communicated to the Society in accordance with Mr Sang's wishes, and is printed without alteration or addition.

On Certain Motions observed by Mr Sang in Cavities of Iceland Spar. By Professor W. Swan, LL.D.

I have received from my friend, Mr Edward Sang, a crystal of Iceland Spar with a letter dated 1st May, in which he writes as follows:—

"In the accompanying little bit of Iceland spar you will find a number of microscopic cavities of various shapes, in which you may perceive a small bubble of vapour, which serves to show the movement of the enclosed fluid."

The glass slide carrying the crystal being placed horizontally on the stage of a microscope, if "you bring a piece of metal, say a coin, gradually until its edge come almost into the field of view, you will see all the bubbles take the (apparently) opposite sides of their cavities: that is to say, the metal repels the fluid. On inclining the microscope the bubbles take the tops" of their cavities, and "you will find that the repulsion exceeds gravity in intensity. I have only found this repulsion with metals: oxides and sulphurets have no action, and each metal has its own specific repulsion. Silver is more active than lead, and, if I mistake not, also than gold. Mercury has little or no effect."

To-day (5th May) I had no difficulty in verifying Mr Sang's result as to the motion of the vapour bubbles when a coin touching the Iceland spar was brought near the fluid cavities, but the experiments I was thereafter induced to make lead to conclusions in some respects differing from those which he has obtained.

Having placed his specimen of Iceland spar on the stage of an excellent Ross's microscope belonging to the United College, and using a one-inch object glass, I saw distinctly the motion of the vapour bubbles, when a florin piece *taken out of my pocket* was brought up, touching the surface of the spar so to come into the field of view, and nearly to cover the fluid cavity observed. The apparent effect was the *attraction* of the vapour bubble, which always ran to the side of the cavity nearest to the edge of the coin. I could distinctly mark the tendency which the bubble exhibited to run in a direction normal to the edge of the piece of metal.

Before having tried any experiments, and while meditating on Mr Sang's letter, I could not help concluding that most probably

heat would prove to be the agent which caused the curious motions which he had observed. I therefore placed the coin outside the window to be cooled in the east wind, and the rain which was falling plentifully. I found that the cold coin caused no sensible motion of the bubbles. I next heated the coin in a spirit lamp flame as hot as I could conveniently handle it. Its energy in moving the bubbles was now so greatly increased, that in some trials rapid motions were observed while the coin was still *out* of the field of view.

Seeing that the bubbles thus moved towards the heated side of their cavities, I concluded that they ought to be repelled from a side which was cooled: and to try if such were the case, I cooled a florin piece in a freezing mixture of nitre, sal ammoniac, and water, to a temperature below 0° C. I had now the satisfaction to find that the cold coin, resting on the spar and brought up towards a cavity, sent the bubble away to the remote side of the cavity, just as the hot coin had brought it to the near side.

It is clear, then, that the phenomenon is not due to a repulsion of the liquid in the cavity by the piece of metal, but is a consequence of the passage of a heat current through the liquid, the bubble always moving in a direction opposite to that in which heat is flowing.

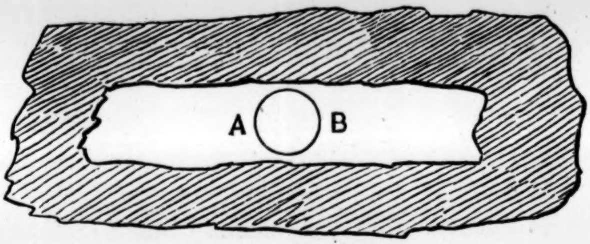
I found that metals possess no specific property in causing these motions. The bubbles moved on the approach of a silver coin or a copper wire. But similar motions were readily obtained when, instead of these, were substituted a heated rod of glass, a slender thin test tube containing hot water, or a piece of shellac moulded into a pencil shape, and still hot from the flame employed to soften it. All these substances—glass, water in the thin tube, and shell-lac—when cooled in the freezing mixture, repelled, or seemed to repel, the bubble, just as when heated they had attracted, or seemed to attract it. The temperatures in these experiments were not accurately observed, but they must have been as follows:—Coins taken from my pocket would be hotter than the air of the room, which was 10° C. or 50° F. The coins being of silver, an excellent conductor of heat, would, when held in the hand, be hotter than the spar lying on the microscope stage in air at 10° C. The coins taken from the flame were as hot at first as could well be handled, and

therefore hotter than the spar. The freezing mixture at the conclusion of the experiment had still a temperature of $-5°$ C. or 23° F.

I found that the direction of motion of the bubbles was the same whether a heated copper wire was held above or below the Iceland spar, with the slide resting horizontally on the stage of the microscope. I fully verified Mr Sang's statement regarding the motion of the bubbles when the microscope is inclined. Placing its tube horizontally, so that the face of the stage and the glass slide were vertical, the bubbles, of course, all rose to the tops of their cavities. A hot copper wire or silver coin touching the surface of the spar at a point on a lower level than that of one of the cavities, instantly drew the bubble down to the bottom. The motion in a vertical plane with a tolerably hot wire seemed almost as brisk as it had been in a horizontal direction, so as to indicate that the effect of hydrostatic pressure, due to gravity, on the minute bubble was trifling as compared with the action set up by the heat current.

Considering the enormous dilatability by heat of liquids, which, under ordinary conditions of temperature and pressure, are permanent gases, I at first thought the motions of the bubbles might be due to currents caused by unequal heating of the liquid on opposite sides of the cavities. The heat flowing from a piece of metal brought near a cavity would cause dilatation of the liquid on the nearer side. A current would then evidently flow along the upper surface of the cavity away from the heated metal, and carry a bubble resting at the top in that direction. But this is precisely the reverse of the motion actually observed; so admitting, as we can scarcely doubt, the setting up of a current due to unequal heating, there must be some other and more energetic action at work, causing a real or apparent motion against any such current; and this I take to be rapid evaporation and condensation of the liquid on opposite sides of the bubble. Suppose A B to be a bubble floating in a cavity through which a heat current passes in the direction A B. A state of equilibrium of the bubble is then evidently impossible. Liquid will evaporate from the hotter side A of the bubble, and vapour will condense into liquid on the colder side B. The liquid surface A will then, by continual loss, travel in the direction B A, and the surface B will by continual gain follow A in the same direction; so

that there will be an apparent motion of the bubble towards the hotter end of the cavity,—an *apparent* motion only, for in reality it is not one and the same mass of vapour which is travelling through the liquid. Any such identical bubble has only a momentary existence. It is continually being changed into a new bubble in a




new position by the accretion of vapour on the side A, and by the restoration of vapour to the liquid state on the side B; and the change of place of the existing bubble is in the direction from B to A, or in a direction opposite to that of the heat current. Such a motion, it is scarcely necessary to remark, agrees with that which is actually observed.

The action thus set up in a vapour bubble is precisely that which takes place in Wollaston's cryophorus, where vapour, rapidly generated at the hotter, is recondensed into liquid and frozen at the colder end of the apparatus. Suppose a cryophorus to consist simply of a cylindrical tube placed vertically and cooled by a freezing mixture at its upper end. The cavity occupied by vapour will then suffer a continual displacement downwards; for the surface of the water, which is its lower boundary, is being depressed through loss by evaporation, while the glass at the top, which was at first its upper boundary, is becoming coated with ice of constantly increasing thickness. The downward displacement of the cavity of such a cryophorus may serve to illustrate that of a bubble in a liquid heated from below. But it may seem that any movement thus produced would be far too slow to displace a bubble downwards, which was rising freely through a liquid. In considering such an objection to the proposed explanation of the motions of bubbles in the cavities of crystals, when these motions take place vertically, or otherwise than in a horizontal direction, it is to be borne in mind,

—first, that the upward motion through a liquid of a *microscopic* bubble must necessarily be very slow, even although under a high magnifying power it may seem otherwise; and next, that in space containing only vapour of a liquid in contact with the liquid itself, evaporation and recondensation may proceed with excessive rapidity. The action of Wollaston's cryophorus, to which reference has just been made, and Dalton's experiments on vapours, made by passing liquids up into a Torricellian vacuum, alike exhibit the facility with which vapours form and recondense in spaces void of gases which are permanent at the existing temperature. Add to these considerations the information derived from the experiments of Cagniard de la Tour, Faraday, and Andrews, as to the enormous celerity with which substances pass from the liquid to the gaseous, or from the gaseous to the liquid condition, when near their critical temperatures, which for different substances range probably between the very remote limits 773° and −166° Fahrenheit, and the explanation which I have ventured to propose of the motion of a vapour bubble in a liquid conveying a heat current becomes sufficiently feasible.

I have to-day been at some pains to verify the result obtained yesterday, namely, that a piece of metal at the same temperature as the Iceland spar has no power to move the globules of vapour in the fluid cavities. Placing a shilling on the microscope stage beside the crystal, I left it for about ten minutes. Then holding it in forceps to avoid heating it by the hand, I moved it up into the field touching the spar, and so as almost to cover a fluid cavity from view. No motion of the bubble ensued. But, on putting my finger on the top of the shilling, by-and-by the bubble began to move, and slowly but steadily crossed the cavity towards the shilling. The same experiment was repeated with a bit of sheet lead about an inch square and 0·08 inch thick, with precisely the same result. I do not find lead notably less active than silver; but the experiments made were necessarily too hasty and imperfect to settle the point as to whether any difference exists. The relative thermal conductivities of silver and lead being in air as 100 to 8·5, according to Wiedemann and Franz's experiments, we might expect, when heat was conducted from the hand into the crystal through a piece of metal, that silver would produce more energetic effects than lead. May the effects be due, in part at least, to radiant heat, the

liquid in the cavities being possibly less diathermanous than the Iceland spar, and absorbing the heat transmitted to it by radiation through the crystal?

In order to try if the motion of a vapour bubble could be exhibited on a larger scale, I made use of a hermetically sealed tube containing liquefied sulphurous acid (sulphur dioxide) which I had some time ago prepared to show the high dilatability by heat of that liquid. When the tube was placed horizontally the void space, like the bubble of a spirit level, was about 15 inches long; and I found that its extremity moved towards the point where a piece of heated brass was applied to the tube. I then nearly filled a tube with ether made from methylated alcohol; and after heating the top, so as to vaporise the ether and expel the air, I hermetically sealed the tube. Placing the tube horizontally, the vapour bubble is about 0·3 of an inch long; and when a finger is put on the tube about 0·25 of an inch from the bubble, in a little while the bubble moves towards the finger with a rapidly accelerated motion, and places itself in a position of stable equilibrum under the finger, about which it slightly oscillates even after the finger is removed from the tube. A piece of metal too hot to be touched acts still more energetically.

I have thought it proper to note that the ether I used had been made from methylated alcohol, because in exhibiting as a lecture experiment Dalton's method of measuring vapour tensions, I have found that ether made from methylated alcohol seems to show a higher vapour tension than that of ether as determined by Regnault. This is probably due to the presence of some other substance more volatile than common or diethyl ether, possibly to a portion of dimethyl ether whose boiling-point is so low as $-21°$ C.

UNITED COLLEGE, ST ANDREWS,
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### 5. Preliminary Note on the sense of Rotation and the Function of the Semicircular Canals of the Internal Ear. By Professor A. Crum-Brown.

As far as I am aware, the sense of rotation has not hitherto been recognised either by physiologists or by psychologists as a distinct sense, but a little consideration and a few experiments seem to me to be enough to show that it really is so. By means of this sense we are able to determine—*a*, the axis about which rotation of the head takes place; *b*, the direction of the rotation; and *c*, its rate.

In ordinary circumstances we do not wholly depend upon this sense for such information. Sight, hearing, touch, and the muscular sense assist us in determining the direction and amount of our motions of rotation, as well as of those of translation; but if we purposely deprive ourselves of such aids we find that we can still determine with considerable accuracy the axis, the direction, and the rate of rotation. The experiments that I have made with the view of determining this point were conducted as follows: a stool was placed on the centre of a table capable of rotating smoothly about a vertical axis; upon this the experimenter sat, his eyes being closed and bandaged; an assistant then turned the table as smoothly as possible through an angle of the sense and extent of which the experimenter had not been informed. It was found that, with moderate speed, and when not more than two or three complete turns were made at once, the experimenter could form a tolerably accurate judgment of the angle through which he had been turned. By placing the head in various positions, it was possible to make the vertical axis coincide with any straight line in the head. It was found that the accuracy of the sense was not the same for each position of the axis in the head, and further, that the minimum perceptible angular rate of rotation varied also with the position of the axis.

The sense of rotation is, like other senses, subject to illusions, rotation being perceived where none takes place. Vertigo or giddiness is a phenomenon of this kind.

When, in the experiments just mentioned, rotation at a uniform

angular rate is kept up for some time, the rate appears to the experimenter to be gradually diminishing; if the rotation be then stopped, he experiences the sensation of rotation about the same axis in the opposite direction. If the position of the head be changed after the prolonged rotation has been made, the position of the axis of the apparent rotation is changed, remaining always parallel to a line in the head which was parallel to the axis of the real rotation. The readiness with which this *complementary apparent rotation* is produced is not the same for each axis. In such experiments, as long as the eyes are shut, and the axis of rotation kept vertical, a sensation of giddiness is not experienced. That sensation appears to be caused by the discordance between the testimony of the sense of sight and that of the sense of rotation.

It is obvious that this sense must have a peripheral organ physically constituted so as to be affected by rotation, and that it must be such as to receive different impressions when the axis, direction, or rate of rotation is changed. These impressions must be transferred to the ends of afferent nerves, and by these nerves conducted to a central organ.

The semicircular canals of the internal ear are eminently fitted by their form and arrangement to act as the peripheral organ of this sense. I shall consider first the action of one semicircular canal, and for simplicity suppose that there is only one. Starting from rest, let us suppose rotation of the head to take place about an axis at right angles to the plane of the canal. The bony canal, being part of the skull, of course shares in this rotation, but the perilymph lags behind, and thus the membranous canal, which floats in the perilymph, does not immediately follow the motion of the bony canal, but, as the membranous canal is continuous at both ends with the utricle, the relative motion of the bony and the membranous canal must produce a pulling or stretching of the forward end of the membranous canal. If this is the end at which the ampulla is situated, such stretching will necessarily move the terminal nervous organs in the ampulla, and may reasonably be expected to stimulate the nerves. This stimulus will no doubt be greater the stronger the pull, *i.e.*, the more rapid the rotation. We should thus with one semicircular canal have the means of perceiving, and of estimating the rate of, rotation in

*one* direction about *one* axis. But we have *six* semicircular canals, three in one ear and three in the other, and these are arranged in pairs—the two exterior being nearly in the same plane, and the superior in one ear being nearly parallel to the posterior in the other. We have thus a system of three rectangular axes, each axis having two semicircular canals at right angles to it,—one influenced by rotation in one direction about the axis, the other by rotation in the opposite direction. Any rotation whatever of the head can be resolved into three rotations, one about each of the said three rectangular axes, and will thus in general affect three ampullæ. If the ampullæ affected are known, and the amount of pull at each is known, the axis about which rotation takes place and the rate of the rotation can be deduced.*

I am at present engaged in making measurements and experiments in reference to this inquiry, and hope before long to lay a more complete account of the various phenomena before the Society.

* When rotation has continued for some time, friction of the periosteum of the bony canals against the perilymph, and fluid friction in the perilymph, gives to the perilymph, and, of course, also to the membranous canal, the same rotation as the bony canal has; the perception of rotation will thus cease. If we now stop the rotation of the head the bony canal stops, but the perilymph and the membranous canal move on, and a pull takes place at the opposite ends of the semicircular canals, causing a perception of rotation round the same axis in the opposite direction.

The members of the three pairs of semicircular canals are not always accurately parallel to each other, and in some animals the three axes are not accurately at right angles, so that in the most general case we have two systems of co-ordinates, not necessarily rectangular, which we may call $x$, $y$, $z$, and $\xi$, $\eta$, $\zeta$—each of these six axes having an organ capable of being influenced by rotation about the axis *in one direction*. But in all cases, as far as I know, these six axes and the corresponding organs are so placed that a different set of impressions will be produced by each form of rotation, that is, by each combination of axis, direction, and rate.